Replica Exchange Light Transport on Relaxed Distributions

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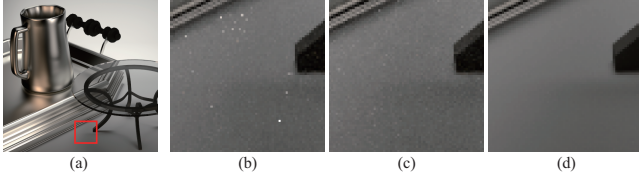
Figure 1: *(a) Reference image. (d): An enlarged view of the region marked red in (a). (b) and (c) show the enlarged rendered results for the same region, using [Kelemen et al. 2002] and our method, respectively. (b) and (c) are rendered with the same computation time.*

1 Introduction

Developing a robust method for computing global illumination is a challenging problem. A Markov chain Monte Carlo (MCMC) method, like [Jakob and Marschner 2012], samples the light path space with a probability proportional to the per-path contribution, by successively mutating path samples (e.g., perturbing a reflection direction). In practice, a path sample could get stuck in a high energy peak for multiple mutations, resulting in a bright spot artifact. To resolve this problem, we present a new unbiased rendering framework based on a replica exchange technique [Kitaoka et al. 2009], a variant of MCMC technique. A replica exchange technique incorporates a set of different distributions. We propose to introduce a set of relaxed distributions, which are beneficial for reducing the chance of getting stuck.

2 Our Framework

We use multiple distributions f_k (Figure 2), where k is a level, and assign a path sample X_k to each of these distributions. Each path sample X_k (a random variable in the sample space corresponding to a light path $\bar{\mathbf{x}}$) is successively mutated to obtain a sequence of the sample $\{X_k^{(i)}\}$, so that the distribution of $\{X_k^{(i)}\}$ is proportional to f_k. The replica exchange technique can be viewed as a Metropolis technique working on a product sample space $\mathbf{X} = X_0 \times X_1 \times \cdots$ with a distribution $F = f_0 f_1 \cdots$.

For the distributions, we choose f_0 to be proportional to the contribution $I(\bar{\mathbf{x}})$ of light path $\bar{\mathbf{x}}$. For higher levels, we propose to choose f_k to be proportional to $I(\bar{\mathbf{x}}) + \varepsilon_k$, where ε_k is a relaxation constant and is larger for higher level k. As the sample space for the light paths, we consider primary sample space (a high dimensional unit cube defining the parameters of a path) introduced by [Kelemen et al. 2002]. Distributions in higher levels are more relaxed, in the sense that when we consider the normalized versions of $I(\bar{\mathbf{x}}) + \varepsilon_k$, a higher level distribution is closer to the uniform distribution.

We utilize two kinds of mutations: (1) mutating a path sample in each level independently, and (2) exchanging path samples between two successive levels. For (1), the mutation and the acceptance probability are defined similarly to [Kelemen et al. 2002]. For (2), the acceptance probability for exchanging two samples $X_k^{(i)}$ and $X_{k+1}^{(i)}$ in successive levels k and $k + 1$ is defined as

$a(X_k^{(i)} \leftrightarrow X_{k+1}^{(i)}) = min(1, \frac{f_k(X_{k+1}^{(i)})f_{k+1}(X_k^{(i)})}{f_k(X_k^{(i)})f_{k+1}(X_{k+1}^{(i)})})$. Since the peaks in higher levels are more smoothed, the acceptance proba-
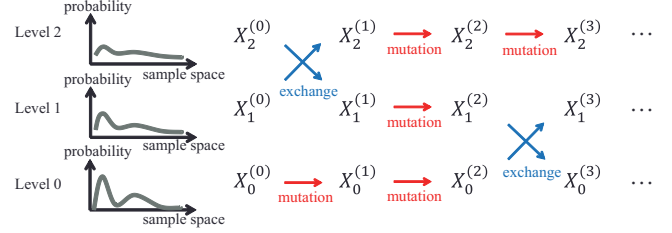

Figure 2: *Illustration of our framework.*

bility for (1) becomes higher for a higher level. Hence, the samples can be more easily mutated across the whole space, through the relaxed distributions.

As X_0 is proportionally distributed according to $I(\bar{\mathbf{x}})$, it can be directly used for rendering the image similar to a Metropolis framework. A sample in any higher level k, can also be used to provide an unbiased estimate of the final image, by multiplying a factor of $\frac{c_k I(\bar{\mathbf{x}})}{I(\bar{\mathbf{x}}) + \varepsilon_k}$, where $c_k = \int_{\mathcal{P}}(I(\bar{\mathbf{x}}) + \varepsilon_k)d\bar{\mathbf{x}}$ is a normalizing factor computed from a uniform sampling of the space (*i.e.*, via large step mutations), and $\mathcal{P}$ is the light path space. (We can regard that estimate as a result of performing importance sampling according to $I(\bar{\mathbf{x}}) + \varepsilon_k$.) Additionally, we combine the estimates from different levels using multiple importance sampling for variance reduction.

3 Results and Future Work

We show the rendered results in Figure 1, together with a comparison against previous method [Kelemen et al. 2002]. With the same computation time, our method can significantly reduce the bright spots. An interesting remark is that although the relaxed distribution is different from the distribution of contributions hence seems to be less efficient in terms of importance sampling, they indeed help improve the overall computation efficiency. Moreover, since the values of a relaxed distribution are always larger than 0, ergodicity is guaranteed even if we only use local perturbations, which is not the case in previous MCMC based rendering techniques. As future work, we would like to theoretically analyze the efficiency of our framework, and to seek for a spatial subdivision structure, where each region has an individual relaxation constant, for further improvement of the efficiency. We also plan to combine our method with [Jakob and Marschner 2012].

References

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